Calculations:

Maximum distance is achieved when the ball is launched 45º above the horizon. At this setting, the initial velocity is the same vertically (v_x) and horizontally (v_y) and relates to exit velocity (|**v**|) through the following equation:

$$|\vec{v}| = \frac{\sqrt{2}}{2}v_x + \frac{\sqrt{2}}{2}v_y = \sqrt{2}v_x = \sqrt{2}v_y$$

The ball should follow a parabolic path starting at 1.25m elevation and should land when it reaches 20m. If y is the highest elevation the ball reaches, t_1 is the time it takes to reach this elevation, t_2 is the time after that it takes to land, t is the total time, and g is the gravitational constant (~9.81m/s^2), the following 6 equations can be used to describe the ball's path:

$$\begin{aligned}
&[1] \quad v_x = v_y \\
&[2] \quad v_x t = 20\text{m} \\
&[3] \quad v_y + g t_1 = 0 \\
&[4] \quad y = v_y t_1 + \tfrac{1}{2} g {t_1}^2 \\
&[5] \quad -(y + 1.25\text{m}) = \tfrac{1}{2} g {t_2}^2 \\
&[6] \quad t_1 + t_2 = t
\end{aligned}$$

Solving these equations, we find $v_x = v_y = 9.61$m/s. Therefore, for the ball to reach 20m (ignoring air resistance), it must exit the launcher at |**v**| ~ 13.6m/s.

If we assume exit velocity is the same as the tangential velocity of the launcher's wheels, we can use the DC motor equations to determine the corresponding torque for that velocity. This effectively tells us the maximum torque the motors can experience during launch while still achieving a launch distance of 20m. Ideally, this number will be a bit higher than the actual experienced torque. This way the ball will still reach the desired launch distance after accounting for air resistance, variation in ball size, and other factors we didn't account for.

Assuming the motor is at steady state, we can use the following equation, where k_t is the torque constant.

$$\omega = \frac{1}{k_t}V - \frac{R}{{k_t}^2}\tau \qquad k_t = \frac{\tau}{I}$$

The 1/4 HP DC motor (link at bottom) specifies a nominal torque of 8.8in-lb = .994Nm and a nominal current of 21A. This gives us a motor constant k_t = .0473Nm/A. Rearranging the the equation to solve for R we get:

$$R = \frac{k_t V}{\tau} - \frac{k_t^2 \omega}{\tau}$$

A nominal angular velocity of 1,800RPM = 188rad/s is also given and the motor will operate at 12V, so R = .148Ω. Now we can rearrange the equation again to solve for the critical torque needed for a 20m launch.

$$\tau = \frac{k_t V}{R} - \frac{k_t^2 \omega}{R}$$

Since the wheels are 6in = .152m and a speed of ~13.6m/s is needed, angular velocity at the critical torque should be ω = 89.5rad/s. Using this value, we get a critical torque τ_{crit} = 2.48Nm. This corresponds to a 16.3N or 3.66lb force applied against the rotation of each wheel during launch. Considering rolling resistance is generally at least an order of magnitude lower than its corresponding normal force, this is a reasonable value.

Ultimately, several motors of varying power should be tested before constructing a full prototype to determine which has the best performance, but from these calculations we know a 1/4 HP motor is at least a good place to start.

Motor:
https://www.grainger.com/product/LEESON-DC-Permanent-Magnet-Motor-48ZG47